Stat 534: formulae referenced in lecture, week 7: Open population models, part 2

Assumptions of "usual" CJS

- No heterogeneity in time-specific capture probability, so like Mt
- All animals alive at t have same ϕ_t
- no loss, missed, or incorrectly recorded marks
- emigration is permanent
  - So confounded with survival, not P[capture]
- fates are independent
- instantaneous sampling periods and immediate release. Why?
  - Imagine short-term study.
    Period 1 is days 1-4, Period 2 is days 8-11.
  - Compute ϕ_1 =P[survive period 1 - period 2].
    Assume same for animals caught:
  - Days 4 and 8 (4 day survival)
  - Days 1 and 11 (10 day survival)
  - $\Rightarrow$ heterogeneity in ϕ_1 if per day survival is constant
  - Not a problem if mortality only happens on day 7 (but I can't imagine a biological reason for this)
- Many of these can be relaxed

Population size estimates from CJS

- Add component for 1st capture process to lnL
  - Links N_i, p_i, i indexes time
  - $\Rightarrow$ JS
  - Kendall 1985 has the modern version
- Horvitz-Thompson

  - $\hat{N}_i = \sum_j 1/p_{ij}$.
    i indexes time
    j indexes individuals
  - In simple time only model, $\hat{N}_i = n_i/p_i$

- Variance of the HT estimate? - details in readings

  - A very useful expression for variance

$$\text{Var } \hat{N}_i = \text{Var E } \hat{N}_i|p_i + \text{E Var } \hat{N}_i|p_i$$

  - Links marginal variance (Var $\hat{N}_i$) to conditional moments, E $\hat{N}_i|p_i$ and Var $\hat{N}_i|p_i$
  - $\Rightarrow$ terms involving E $[1/p_{ij}]$, Var $[1/p_{ij}]$, and Cov $[1/p_{ij}, 1/p_{ij'})]$
  - Cov is the covariance between two individuals (j and j') at same time (i)
  - Use 1st order Taylor series expansions

- What you need to know about the variance estimator:

  - Different versions are available
  - What do you do with the Cov term?
    include it or not?
  - Without is very much easier to calculate
  - Cite your source or software!

Pradel reverse time estimators

- see hand-written notes

- Notation

  - γ_{i+1}: "seniority" parameter
  - fraction of population at time $i+1$ also alive at time i

- # Births between i and $i+1 = N_{i+1}(1 - \gamma_{i+1})$

- Pradel gives a direct estimate of birth rate

  - Included in the likelihood
  - Not a derived parameter

- CJS estimates survival of marked individuals
- Pradel estimates birth rate for individuals that are or will be marked

Pradel temporal symmetry model: estimate pop growth rate

- Combine CJS (forward time) and Pradel (reverse time)
- $\Rightarrow$ direct estimate population growth rate

$$\frac{N_{i+1}}{N_i} = \lambda_i = \frac{\phi_i}{\gamma_i}$$

- Explanation/derivation in hand-written notes